ORIGINAL ARTICLE

Boundary Layer Flow over a Moving Horizontal Plate in a Moving Fluid with the Presence of Thermal Radiation

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Abstract
Analysis of steady laminar boundary layer flow past a moving plate in a viscous incompressible fluid with the presence of thermal radiation has been presented in this paper. The cases when the plate and the fluids moves in the same direction and reverse to each other with species concentration are considered for the present study. The governing partial differential equations are transformed into ordinary differential equations using similarity transformations, which are more convenient for numerical computations. The transformed ordinary differential equations are then solved numerically by the Keller box method. Some numerical results obtained are compared with previously reported cases available from the literature and they are found to be in a good agreement. In addition to this, a parametric study is performed in the investigation to illustrate the influence of various parameters on the velocity, temperature and concentration profiles.

Keywords: *Boundary layer, moving plate, mass diffusion, Keller box method, velocity ratio, thermal radiation*

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Nomenclature

C_w	species concentration at the wall
C_∞	species concentration far from the surface
c_p	specific heat capacity
F	dimensionless velocity
f	dimensionless stream function
G	dimensionless temperature
H	dimensionless concentration
k	thermal conductivity
k^*	mean absorption coefficient
P_r	Prandtl number
Sc	Schmidt number
T	temperature
T_∞	temperature of the fluid far away from the wall
T_w	temperature at the wall
r	ratio of stream velocity to composite reference velocity
U	composite reference velocity
u	velocity component in x-direction
U_w	moving plate velocity
U_∞	free stream velocity
v	velocity component in y-direction
q_r	radiative heat flux

Greek symbols

α	thermal diffusivity
ν	kinematic viscosity
ρ	density
η	transformed variable
ψ	stream function
σ^*	Stefan-Boltzmann constant

INTRODUCTION

Investigation of steady flow of incompressible fluid flow has attracted considerable attention in recent years due to its vital role in numerous engineering applications, industrial manufacturing processes such as hot rolling, wire drawing, glass-fiber, drawing of plastic films, metal and polymer extrusion and metal spinning. Furthermore, study of boundary layer behavior over a moving surface in a parallel stream has important practical applications such as in aerodynamic extrusion of plastic sheets, the cooling of an infinite metallic plate in a cooling bath, the boundary layer along material handling conveyors, the boundary layer along a liquid film in condensation processes, paper production and others. On the other hand, convective heat transfer with radiation studies are very important in processes involving high temperatures such as gas turbines, nuclear power plants, and thermal energy storage and so on. But very little is known about the effects of radiation on the boundary layer.

Investigators encounter actually a wide variety of challenges in obtaining suitable algorithms for computing flow and heat transfer of viscous fluids numerically and analytically. The behavior of steady boundary layer flow over a moving flat surface with constant velocity was first presented by Sakiadis (1961). Significant differences were found between this behavior and the behavior of the boundary layer in a moving fluid over a steady flat surface, considered by Blasius (1908). Bataller (2008) investigated classical Blasius flat-plate flow in fluid mechanics in the presence of thermal radiation showing the Prandtl number tends to reduce the thermal boundary layer thickness along the plate. The boundary layer flow on a moving flat surface to a parallel free stream, and the case when the surface and the free stream move in the same direction with constant velocity was analyzed by Abdelhafez (1985). Furthermore Afzal (1993) analyzed the case when the wall and the free stream move in opposite directions, and showed that dual solutions exist. In addition Tsou (1967) made an experimental and theoretical treatment of the boundary layer flow on a continuously moving surface. They concluded that measurements of the laminar velocity field are in excellent agreement with the analytical predictions. Consequently they pointed out that analytically describable boundary layer on a continuous moving surface is a physically realizable flow. Ishak et al (2009) scrutinized the boundary layer flow on a fixed or moving surface parallel to a uniform free stream with constant surface heat flux investigating dual solutions exist when the sheet and the free stream move in the opposite directions. Patil et al. (2009) considered the steady, laminar mixed convection flow over a continuously moving semi infinite vertical plate due to the combined effects of thermal and mass diffusion in the presence of internal heat generation or absorption and an n^{th} order homogeneous chemical reaction between the fluid and the diffusing species. They solved numerically the coupled nonlinear partial differential equations using implicit finite difference scheme in combination with quasilin earization and analysed the profile of velocity, temperature and concentration with different parameters. Anilkumar (2011) obtained the non-similar solution of an unsteady laminar mixed convection on a continuously moving vertical plate by taking into account the effect of viscous dissipation for both accelerating and decelerating free-stream velocities. He demonstrated that the skin friction and heat transfer coefficients are significantly affected by the time dependent free-stream velocity distributions.

A numerical study of the momentum and heat transfer of an incompressible fluid past a parallel moving sheet based on composite reference velocity U was examined by Cortell (2008) and found that the direction of the wall shear changes in an interval $0 \leq r \leq 1$ and an increase of the parameter r yield an increase in temperature. Additionally, same authors (Cortell (2008) analyzed the effect of thermal radiation on the laminar boundary layer about a flat-plate in a uniform stream of fluid (Blasius flow), and about a moving plate in a quiescent ambient fluid (Sakiadis flow) both under a convective surface boundary condition depicting that increase in Prandtl number Pr and the radiation parameter N_R tend to reduce the fluid temperature.

Lin and Haung (1994) analyzed a horizontal isothermal plate moving in parallel or reversibly to a free stream where similarity and non-similarity equations are used to obtain the flow and thermal fields. Further, Muthucumaraswamy and Janakiraman (2006) investigated the effect of thermal radiation on unsteady free convective flow over a moving vertical plate with mass transfer in the presence of magnetic field. They considered gray, absorbing-emitting radiation but a non-scattering medium fluid and used Laplace-transform technique to solve the problem.

Cortell (2007) gave suitable solutions to the classical Sakiadis flat-plate flow in an in compressible fluid in the presence of thermal radiation. Makinde (2011) applied similarity method to investigate the effect of an exponentially decaying internal heat generation on a boundary layer flow over a moving vertical plate with a convective boundary condition. Recently, Bachok and Ishak (2012) studied steady boundary layer flow of a viscous fluid on a moving flat plate in a parallel free stream with variable fluid properties using Keller-box method. Palani et al (2013) have studied a laminar free convection heat transfer flow past a semi-infinite vertical cone with a variable surface heat flux by implicit finite difference scheme of the Crank-Nikcolson type depicting that a greater viscus dissipation of heat causes a rise the local skin friction.

Earlier works have used the momentum and energy equations to investigate certain physical parameters for a problem of flow over a moving horizontal plate in a moving fluid. Now the present study is considering the thermal radiation and the species diffusion in the boundary layer flow to analyze the effects of physical parameters: the ratio r of the fluid velocity U_∞ to the composite reference velocity U , Prandtl number Pr, radiation parameter N_R and the Schemidit number Sc on velocity, temperature and diffusion of species. The governing partial differential equations are transformed into ordinary differential equations using similarity transformations, which are more convenient for numerical computation. The transformed ordinary differential equations are then solved numerically by the Keller box method. For further knowledge refer the books by Cebeci (1984) and Na (1979).

Mathematical Formulation

Consider a plane surface moving with constant velocity U_w in an incompressible viscous fluid of uniform stream velocity U_∞. The plate moves in parallel or reverse to the free stream velocity in x-direction. The x-axis is taken to be along the plate and the y-axis normal to it. The physical properties of the fluid are assumed to be constant. We assume also that the flow is laminar and the surface is maintained at constant temperature T_w and the concentration of the diffusing species at the wall is constant C_w. Further, the surface and the free stream are at the same temperature or with small temperature difference so that the buoyancy effect on flow is negligible. Following Afzal (1993), taking into account the thermal radiation term in the energy equation and using Boussinesq approximation invoked for the fluid properties, the governing equations for the boundary layer flow are:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \qquad (2.1)$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \nu\frac{\partial^2 u}{\partial y^2} \qquad (2.2)$$

$$u\frac{\partial T}{\partial X} + v\frac{\partial T}{\partial y} = \propto \frac{\partial^2 T}{\partial y^2} - \frac{1}{\rho c\rho}\ \frac{\partial qr}{\partial y}, \tag{2.3}$$

$$u\frac{\partial c}{\partial x} + v\frac{\partial c}{\partial y} = D\frac{\partial 2_C}{\partial y^2} \tag{2.4}$$

with boundary conditions $u = u_w$, $v = 0$, $T = T_w$, $C = C_w$ at $y = 0$ and
$u = u_\infty$,$T = T_\infty$,$C = C_\infty$ as $y \rightarrow \infty$
Applying the following transformations:

$$\eta = y\left(\frac{U}{vx}\right)\frac{1}{2}\ ; \quad U = u_{\infty + u_{w};} \qquad {}_{\psi(x,y)=(vUx)}\frac{1}{2}\, f(\eta);\ \ v = -\frac{\partial\psi}{\partial x}$$

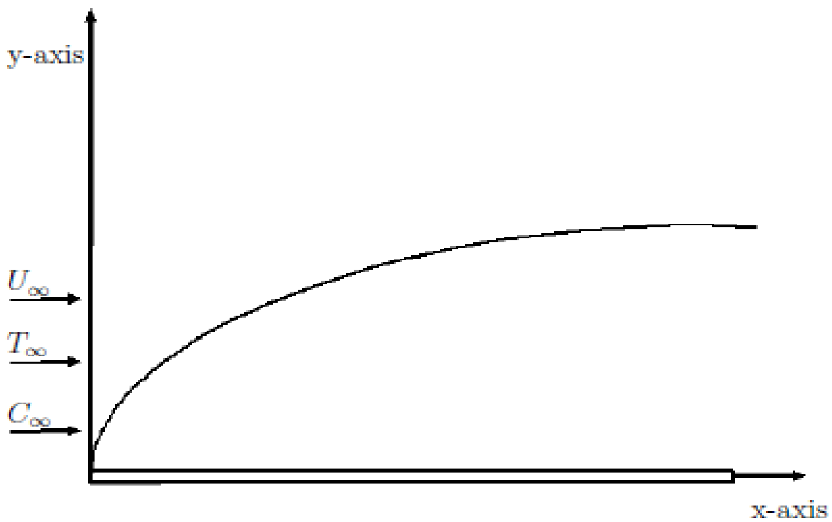

Figure 1: Physical model and coordinate system.

$$u = \frac{\partial\psi}{\partial y};\qquad f' = F;\qquad G(n) = \frac{T - T_\infty}{T_{W} - T_\infty};\quad \mathrm{H}(n) = \frac{C - C_\infty}{C_{W} - C_\infty};$$

$$r = \frac{u_\infty}{U};\qquad u = Uf';\qquad v = \frac{1}{2x}\ (vUx)\frac{1}{2}\ (\eta f' -), Pr = \frac{v}{\alpha};$$

Using the Rosseland approximation [11], the radiative heat flux is simplified as

$$q_{r = -\frac{4\sigma^*}{3k^*}}\frac{\partial T^4}{\partial y} \tag{2.5}$$

where σ^* and k^* are the Stefan-Boltzmann constant and the mean absorption coefficient respectively.
We assume that the temperature differences within the flow such as the term T^4 may be expressed as a linear function of temperature. Thus expanding T^4 in a Taylor series about T_∞ and neglecting higher order terms we get:

$$T^{4\cong 4T_{\infty}^{3}T}-3T_{\infty.}^{4} \quad (2.6)$$

So with regard to equation (2.5) and (2.6), equation (2.3) reduces to

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\left(\alpha+\frac{16\sigma*T_{\infty}^{3}}{3P^{C}p^{k*}}\right)\frac{\partial^{2T}}{\partial y^{2}} \quad (2.7)$$

If we take $N_{R=}\ \frac{kk^*}{4\sigma^{*T}\infty^3}$ as the radiation parameter, (2.7) becomes

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial x}=\frac{\alpha}{\partial y}\ \frac{\partial^{2T}}{\partial y^{2}} \quad (2.8)$$

where $k_{0=}\frac{3N_R}{3N_{R+4}}$

With the similarity variable η and the dimensionless stream function $f(\eta)$, continuity equation is satisfied and equation (2.2), (2.7) and (2.4) respectively reduce to the following ordinary differential equations

$$f'''+\frac{f}{2}f''=0,\quad f(0)=0, f'(0)=1-r, f'(\eta))\to\infty, \quad (2.9)$$

$$G''+\frac{k_{0P_{rf}}}{2}\ G'=0, G(0)=1, G(\eta)\to 0\ as\ \eta\to\infty, \quad (2.10)$$

$$H''+\frac{Scf}{2}\ H'=0,\ H(0)=1, H(\eta)\to 0\quad as\ \eta\ \to\infty, \quad (2.11)$$

Method of Solution

In the present study we treat r as a constant and $-1.5\leq r\leq 1.5$. Observe that if r is set to 1 in equation2.9, it is the classical Blasius flat plate flow problem and for $r = 0$ is that of Sakiadis. The flow problems are solved by the Keller box method. In order to validate our method, we have compared with the earlier work as indicated below in Tables 1 and 2. Computations have been carried out for Pr $(0.7\leq Pr\leq 10)$, $Sc(0.23\leq Sc\leq 50)$, $N_R\,(0.5\leq N_R\leq 10)$, and $\Delta\eta = 0.01$.

The case $0 < r < 1$ is when the plate and the fluid move in the same direction, while opposite direction for $r < 0$ and $r > 1$. If $r < 0$, the free stream is directed towards the positive x-direction while the plate moves towards the negative x-direction. But if $r > 1$, the free stream is in the negative x-direction while the plate moves towards the positive x-direction.

Table 1: Comparison of velocity gradient $F^{'}(0)$ and temperature gradient $G^{'}(0)$ with the earlier work at the moving surface with different Prandtl numbers and $k_0 = 1$

	Backok [5]		Present	
Pr	$-F^{'}(0)$	$-G^{'}(0)$	$-F^{'}(0)$	$-G^{'}(0)$
0.7	0.4437	0.3492	0.443748718	0.3492418986
1	0.4437	0.4437	0.443748718	0.443748718
10	0.4437	1.6803	0.443748718	1.6803274499
100	0.4437	-	0.443748718	5.546202431
1000	-	-	0.4437487181	17.799699308

Table 2: Comparison of velocity gradient $F^{'}(0)$ and temperature gradient $G^{'}(0)$ with the earlier works at the moving surface with $Pr = 0.7$, $k0 = 1$ and constant fluid properties as indicated by Anderson [3].

Authors	$-F^{'}(0)$	$-G^{'}(0)$
Sakiadis [21]	0.44375	-
Tsou [24]	0.444	0.3492
Takhar [23]	0.4439	0.3508
Pop [18]	0.4445517	0.3507366
Pantokratoras [17]	0.4438	0.3500
Present	0.4437487181	0.3492418986

RESULTS AND DISCUSSION

The velocity profile for the parallel flow is depicted in Fig. 2 but Fig.3 and 4 present that of reversible flow. According to Fig. 2 the velocity increases for $r > 0.5$, decreases for $r < 0.5$ and constant for $r = 0.5$. The constant velocity at $r = 0.5$ shows the velocity of the plate and that of fluid is the same. This shows that they have the same contribution to flow of the system. Fig.3 and 4 illustrates that the dimensionless velocity decreases for negative values of r and increases for $r > 1$ in that of reversible flow. The effect of Prandtl number on dimensionless temperature profile without radiation effect with different value of r is presented in Fig.5. The figure demonstrates the temperature profile for different values of Prandtl numbers when $r = 0.1$ and $r = 0.9$.

From the figure the authors concluded that as the Prandtl number increases there is a decrease in temperature. But increasing the value of r increases the temperature when Prandtl number is constant and the flow is in the same direction. Effect of radiation parameter N_R on temperature for different values of velocity ratio r and Prandtl numbers are shown in Fig.6 and 7. The figures depict that the dimensional temperature profile G increases as the radiation parameter N_R decreases. To elucidate more, as the mean absorption coeffcient k^* decreases the expression $\frac{\partial q_r}{\partial y}$ increases. That means the rate of thermal radiation transformed to the fluid increases as accordance with an increase of fluid temperature.

Consequently Fig.7 again reveals that as r increases the temperature increases keeping the Prandtl number $P\ r$ and radiation parameter N_R constant. Physically this means that the non-dimensional temperature is significant when the fluid's velocity is greater than that of the plate with other parameters remain the same.

Fig.8 and 9 express effect of velocity ratio r and Schmidt number Sc on the non-dimensional concentration profile of the species diffusion. Fig 8 shows the species concentration profile for different values of r. The figure reveals that increasing r increases the species concentration with other parameters fixed. This shows when the plate and the fluid move in the same direction, increment in the velocity ratio r enhances the species concentration. This implies that when the free stream velocity is greater than that of the plate's velocity the species concentration becomes large. This may be due to the fact that velocity of the free stream is more but that of the plate's is less and this causes more species to concentrate near the surface of the plate. Figure 9 describes effect of Schmidt number on species concentration. It illustrates that increasing Schmidt number decreases species concentration.

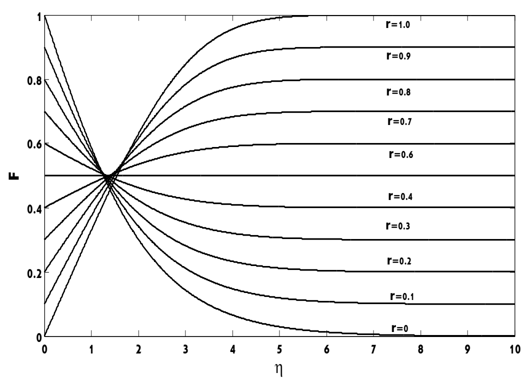

Figure 2: Velocity profile for some values of *r* in an interval [0, 1].

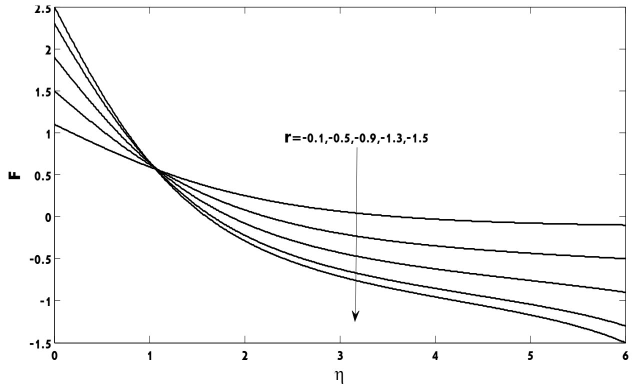

Figure 3: Velocity profile for some values of *r* in an interval [−1.5, 0].

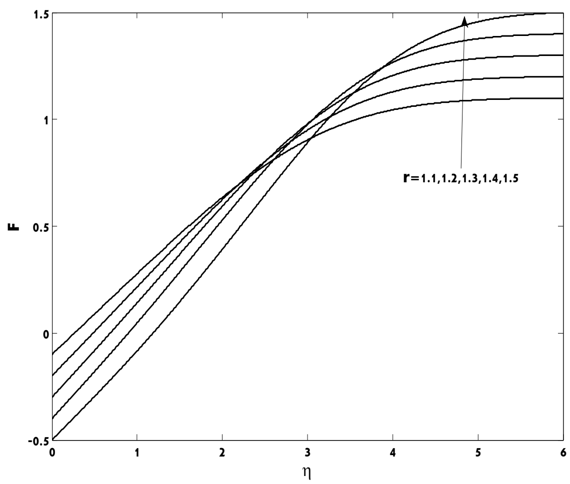

Figure 4: Velocity profile for some values of *r* in an interval [1.1, 1.5].

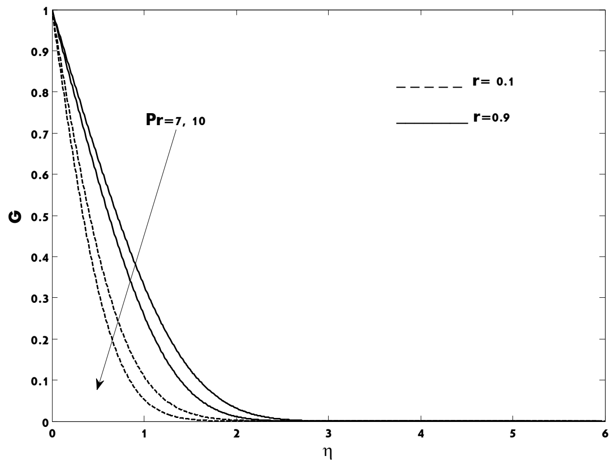

Figure 5: Effect of Prandtl number on temperature without thermal radiation effect.

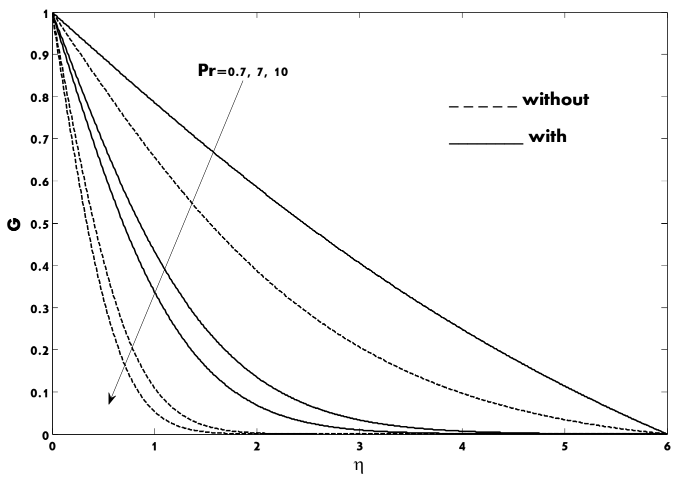

Figure 6: Effect of Prandtl number on temperature with and without thermal radiation when N_R = 0.5r=0.1

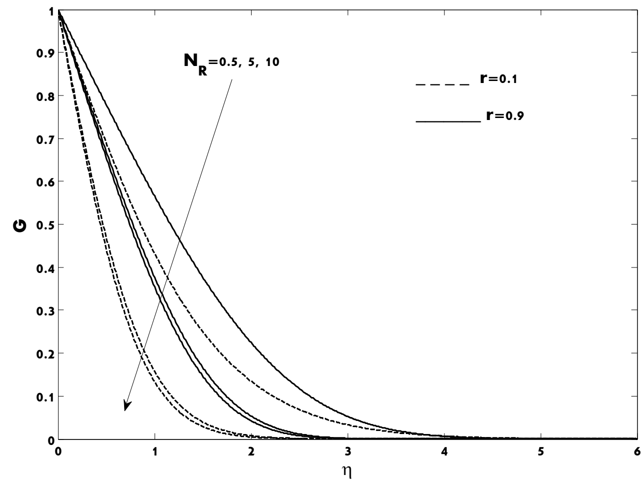

Figure 7: Effect of radiation parameter N_R on temperature. When Pr=7

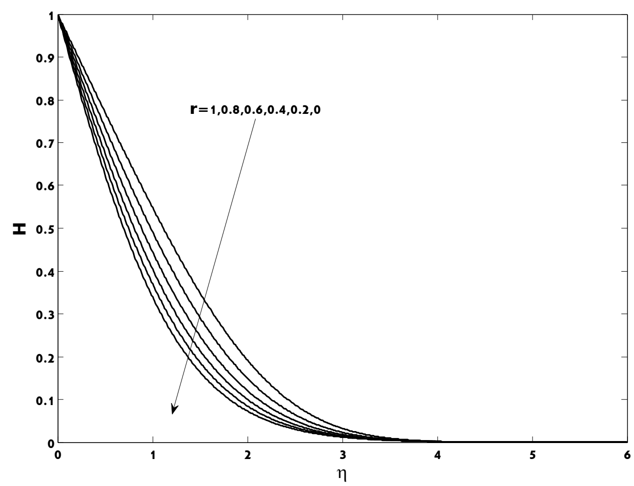

Figure 8: Concentration profile for different values of r.

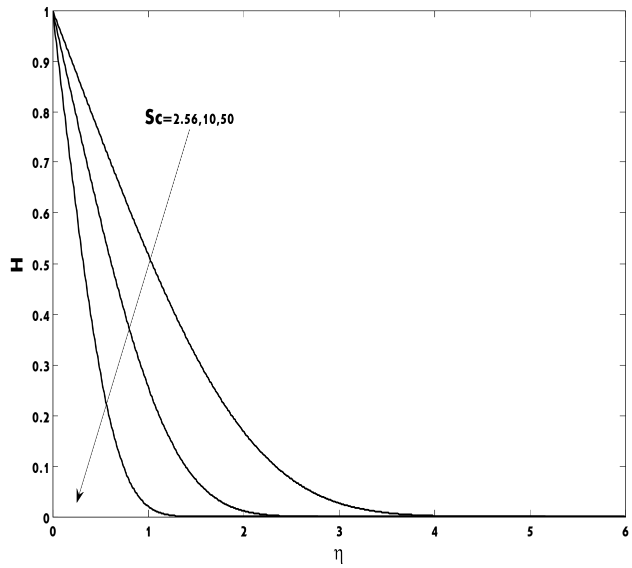

Figure 9: Concentration profile for different values of *r* and *Sc*.

CONCLUSION

We studied the problem of steady laminar boundary layer flow over a moving plate in a viscous incompressible fluid with the presence of thermal radiation. The governing partial differential equations are transformed into ordinary differential equations using similarity transformations. The transformed ordinary differential equations are then solved numerically by the Keller box method. The numerical results obtained are compared with previously reported cases available from literature and they are found to be in good agreement. From the present investigation, we found that:

- The velocity increases for $r > 0.5$, decreases for $r < 0.5$ and constant at $r = 0.5$.
- Raising the Prandtl number tends to reduce the temperature of the fluid and rising of Schmidit number will do the same on species concentration.
- Increasing velocity ratio *r* increases temperature of the fluid with other parameters constant. That means temperature of the fluid is higher if the free stream velocity is greater than that of the plate's velocity keeping other parameters fixed.
- Raising radiation parameter decreases temperature of the fluid.
- Increasing the velocity ration *r* increases the species concentration. This implies that when the free stream velocity is greater than that of the plate's velocity, the species concentration rises.

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